

### Abstract

Some properties of multi-qubit systems interacting with noisy environment is discussed. The amount of the survival entanglement is quantified for the GHZ and W-states. It is shown that the entanglement decay depends on the noise type (correlated or non-correlated), number of interacted qubits with the environment and the initial state which passes through this noisy environment. In general, the GHZ is more fragile than the W-state. The phenomena of entanglement sudden death appears only for non-correlated noise.

## 1 Introduction

Entanglement for two-qubit systems has been investigated extensively in many directions. Quantifying and manipulating entanglement for systems of small dimension have been covered a in wide area of research [1, 2, 3]. However, there are a lot of applications that have been implemented on quantum information and computations by using these systems theoretically and experientially. Quantum teleportation, coding [4] and cryptography represent the more important applications of entanglement. To implement these application with high efficiency, maximum entangled states (MES) are needed. Despite it is possible to generate maximum entangled states, they turn into partially entangled states (PES) due to their interaction with their own environments [11]. Therefore, another process is needed to recover entanglement, called quantum purification, which requires infinite number of pairs to get one MES [5]. So, it is important to investigate the behavior of entanglement in the presences of different types of noise. There are some classes of noise states whose teleportation fidelity can be enhanced if one of the two qubits subject to dissipative interaction with the environment via amplitude damping channel [6].

Recently, Montealegre et. al. [7], have investigated the effect of different types of noise channels on the one-norm geometric discord and showed that the effect of the noise channel can be frozen. Metwally showed that the phenomena of sudden single and double changes and the sudden death of entanglement are reported for identical and non-identical noise [8]. Also, the behavior of a multi-qubit system in the presence of the amplitude damping channel is investigated in [9], where the possibility of performing quantum teleportation with decohered states was discussed. This motivates us to investigate two classes of multi-qubit systems, the GHZ and the W-states, where the effect of the correlated or non-correlated noisy depolarized channel is considered.

The paper is organized as follows: In Sec. 2, we describe the initial state and their evolution in the presence of depolarized damping channel. The behavior of entanglement for the GHZ state is discussed in Sec. 3, while Sec. 4, is devoted for the W-state. Finally we conclude our results in Sec. 5.

## 2 The system

Assume that a source generates a three-qubit state in the form of GHZ or W-state defined as:

$$|\psi_{ini}\rangle = \begin{cases} |\psi_g\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \\ |\psi_w\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle). \end{cases} \quad (1)$$

During the transmission from the source to the users, Alice, Bob, and Charlie, it is assumed that Alice's and Bob's qubits pass through the depolarized channel [11]. This channel transforms the single qubit into a completely mixed state with probability  $p$  and leaves it untouched with probability  $1 - p$ . For a single qubit noise, the operators are given by

$$\mathcal{D}_1^{(j)} = \sqrt{1-p} I^{(j)}, \quad \mathcal{D}_2^{(j)} = \sqrt{\frac{p}{3}} \sigma_x^{(j)}, \quad \mathcal{D}_3^{(j)} = \sqrt{\frac{p}{3}} \sigma_y^{(j)}, \quad \mathcal{D}_4^{(j)} = \sqrt{\frac{p}{3}} \sigma_z^{(j)}, \quad (2)$$

where  $I_i^{(j)}, \sigma_i^{(j)}, i = x, y, z, j = a, b$  and  $c$  are the identity and Pauli operators for the three qubits respectively. In computational basis, these operators can be written as:

$$\begin{aligned} I_i^{(j)} &= |0\rangle\langle 0| + |1\rangle\langle 1|, \quad \sigma_x^{(j)} = |0\rangle\langle 1| + |1\rangle\langle 0|, \\ \sigma_y^{(j)} &= i(|0\rangle\langle 1| - |1\rangle\langle 0|), \quad \sigma_z^{(j)} = |0\rangle\langle 0| - |1\rangle\langle 1|. \end{aligned} \quad (3)$$

We consider that the suggested noise model is of one, two and three sides noisy channel type. For two and three sides noise, we consider that the noise could be correlated or non-correlated [12]. For correlated noise, we mean that the qubit is affected by the same noise at the same time, while for non-correlated noise, the qubits may be affected by different noises at the same time.

The three-qubit system under the depolarizing noise acting on the first qubit of the quantum state  $\rho_{ini}$  is given by

$$\rho^{(fi)} = \sum_{k=1}^4 \left\{ \mathcal{D}_k^a \otimes I^{(b)} \otimes I^{(c)} \rho_{ini} I^{(c)} \otimes I^{(b)} \otimes \mathcal{D}_k^{\dagger(a)} \right\}, \quad (4)$$

where  $\rho_{ini} = |\psi\rangle_{ini}\langle\psi|$  and  $|\psi_{ini}\rangle$  is one of the initial states (1). However, if the first two qubits are affected by the depolarized channel, then the initial state  $\rho_{ini}$  evolves as

$$\rho^{(fco_2)} = \sum_{k=1}^4 \left\{ \mathcal{D}_k^{(a)} \otimes \mathcal{D}_k^{(b)} \otimes I^{(c)} \rho_{ini} I^{(c)} \otimes \mathcal{D}_k^{\dagger(b)} \otimes \mathcal{D}_k^{\dagger(a)} \right\}, \quad (5)$$

for correlated noise, while for non-correlated noise, the final state can be written as  $\rho^{(f)}$

$$\rho^{(fnc_2)} = \sum_{k=1}^{k=4} \sum_{\ell=1}^{\ell=4} \left\{ \mathcal{D}_k^{(a)} \otimes \mathcal{D}_\ell^{(b)} \otimes I^{(c)} \rho_{ini} I^{(c)} \otimes \mathcal{D}_\ell^{\dagger(b)} \otimes \mathcal{D}_k^{\dagger(a)} \right\}. \quad (6)$$

Finally, if the three qubits are affected by the depolarizing channel (2), then for correlated noise, the final state of the initial three qubits (1) evolves as,

$$\rho^{(f_{co3})} = \sum_{k=1}^{k=4} \left\{ \mathcal{D}_k^{(a)} \otimes \mathcal{D}_k^{(b)} \otimes \mathcal{D}_k^{(c)} \rho_{ini} \mathcal{D}_k^{\dagger(c)} \otimes \mathcal{D}_k^{\dagger(b)} \otimes \mathcal{D}_k^{\dagger(a)} \right\}, \quad (7)$$

while for the non-correlated noise it is given by

$$\rho^{(f_{nc3})} = \sum_{k=1}^{k=4} \sum_{\ell=1}^{\ell=4} \sum_{m=1}^{m=4} \left\{ \mathcal{D}_k^{(a)} \otimes \mathcal{D}_\ell^{(b)} \otimes \mathcal{D}_m^{(c)} \rho_{ini} \mathcal{D}_k^{\dagger(c)} \otimes \mathcal{D}_\ell^{\dagger(b)} \otimes \mathcal{D}_m^{\dagger(a)} \right\}, \quad (8)$$

where  $\rho^{(f_{cor})}$  and  $\rho^{(f_{ncr})}$  when  $r = 2, 3$  stands for the final states for correlated and non-correlated noise if 2 or 3 qubits are affected, respectively.

In the following we shall investigate the dynamics of entanglement which is contained in the initial states (1), where we investigate all the possible noises as described above. For this aim, we use the tripartite negativity as a measure of entanglement. This measure states that, if  $\rho_{abc}$  represents a tripartite state, then the negativity is defined as [13],

$$\mathcal{N}(\rho_{abc}) = (\mathcal{N}_{a-bc} \mathcal{N}_{b-ac} \mathcal{N}_{c-ab})^{\frac{1}{3}}, \quad (9)$$

where  $\mathcal{N}_{i-jk} = -2 \sum_{\ell} \lambda_{\ell}(\rho_{ijk}^{T_i})$ ,  $\lambda_{\ell}$  are the negative eigenvalues of the partial transpose of the state  $\rho_{ijk}$  with respect to the qubit "i"

### 3 GHZ-state

#### 1. Correlated Noise:

The initial three-qubit state  $\rho_g = |\psi_g\rangle\langle\psi_g|$ ,  $|\psi_g\rangle$  is given from (1) under the depolarizing noise on Alice's qubit is given by

$$\begin{aligned} \rho_g^{(f)} &= (1-p)\rho_g + \frac{p}{3} \left( |111\rangle\langle 100| - |111\rangle\langle 011| \right) \\ &\quad + \frac{p}{6} \left( |011\rangle\langle 100| + |100\rangle\langle 011| + |100\rangle\langle 100| + |011\rangle\langle 011| \right). \end{aligned} \quad (10)$$

However, if we assume that the first two qubits subject to correlated noise then the final state (5) is given by

$$\rho_g^{(f_{co2})} = \left( \frac{1}{2}(1-p)^2 + \frac{p^2}{18} \right) \rho_g + \frac{p^2}{9} \left( |001\rangle\langle 110| + |110\rangle\langle 001| + |001\rangle\langle 001| + |110\rangle\langle 110| \right). \quad (11)$$

Finally, if it is assumed that all the three qubits are subject to a correlated noise, then  $\rho^{(f_{co3})}$  can be written as

$$\begin{aligned} \rho_g^{(f_{co3})} &= \left( \frac{(1-p)^3}{2} + \frac{p^2}{18} \right) |000\rangle\langle 000| + \left( \frac{(1-p)^3}{2} - \frac{p^3}{54} \right) |000\rangle\langle 111| \\ &\quad + \frac{(1-p)^3}{2} |111\rangle\langle 000| + \left( \frac{(1-p)^3}{2} + \frac{p^2}{27} \right) |111\rangle\langle 111|. \end{aligned} \quad (12)$$

In Fig.(1), we investigate the behavior of entanglement for the three final states (9 – 11). The general behavior displays that the entanglement decays as  $p$  increases. However, these decays depends on the number of the affected qubits with

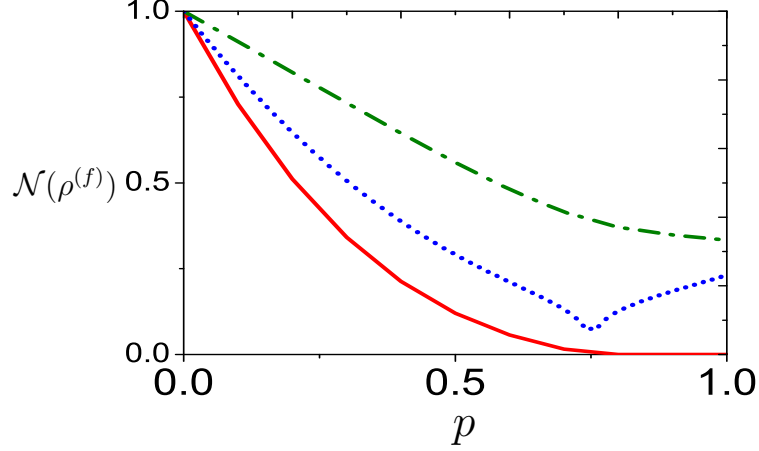


Figure 1: The entanglement  $\mathcal{N}(\rho_g^{(f)})$  of the GHZ state under depolarize channel. The solid and the dot curves for the correlated and non-correlated noise respectively noise.

the depolarized channel. For the state  $\rho_g^{(f)}$ , where only one qubit is depolarized, the entanglement decays smoothly, while for  $p = 1$ , when the strength of noise is maximum, the entanglement doesn't vanish. For the state  $\rho_g^{(f_{co2})}$ , where two qubits are depolarized, the decay rate is larger than that depicted for the state  $\rho_g^{(f)}$ . Although the entanglement vanishes for some specific value of  $p \simeq 0.75$ , it re-increases again for farther values of  $p$ . Finally, the decay rate of entanglement is the largest one if the three qubits are depolarized as shown for the state  $\rho_g^{(f_{co3})}$ . For this state the entanglement decays gradually until vanishes completely at  $p \simeq 0.78$ .

## 2. Non-correlated noise:

Now, we assume that two or three qubits are depolarized in non-correlated way. If only two particle are depolarized then the final sate (6) can be written as

$$\begin{aligned} \rho_g^{(f_{nco2})} = & \mathcal{A}_1 \left( |000\rangle\langle 000| + |111\rangle\langle 111| \right) + \mathcal{A}_2 \left( |000\rangle\langle 111| + |111\rangle\langle 000| \right) \\ & + \mathcal{A}_3 \left( |010\rangle\langle 010| + |101\rangle\langle 101| + |011\rangle\langle 011| + |100\rangle\langle 100| \right) \\ & + \mathcal{A}_4 |001\rangle\langle 110| + \mathcal{A}_5 |001\rangle\langle 001| + \mathcal{A}_6 |110\rangle\langle 110| + \mathcal{A}_7 |110\rangle\langle 001| \\ & + \mathcal{A}_8 |001\rangle\langle 111| + \mathcal{A}_9 \left( |001\rangle\langle 011| + |110\rangle\langle 100| - |110\rangle\langle 011| \right), \end{aligned} \quad (13)$$

where

$$\begin{aligned} \mathcal{A}_1 &= \frac{(1-p)^2}{2} + \frac{p}{3}(1-p) + \frac{p^2}{18}, & \mathcal{A}_2 &= \frac{(1-p)^2}{2} - \frac{p}{3}(1-p) + \frac{p^2}{18}, \\ \mathcal{A}_3 &= \frac{p}{3}(1-p) + \frac{p^2}{9}, & \mathcal{A}_4 &= \frac{p^2}{9} - \frac{p}{3}(1-p), & \mathcal{A}_5 &= \frac{p^2}{18}, & \mathcal{A}_6 &= \frac{p^2}{9} + \frac{p}{6}(1-p), \\ \mathcal{A}_7 &= \frac{p^2}{9} - \frac{p}{6}(1-p), & \mathcal{A}_8 &= \frac{p}{6}(1-p), & \mathcal{A}_9 &= \frac{p^2}{18}. \end{aligned} \quad (14)$$

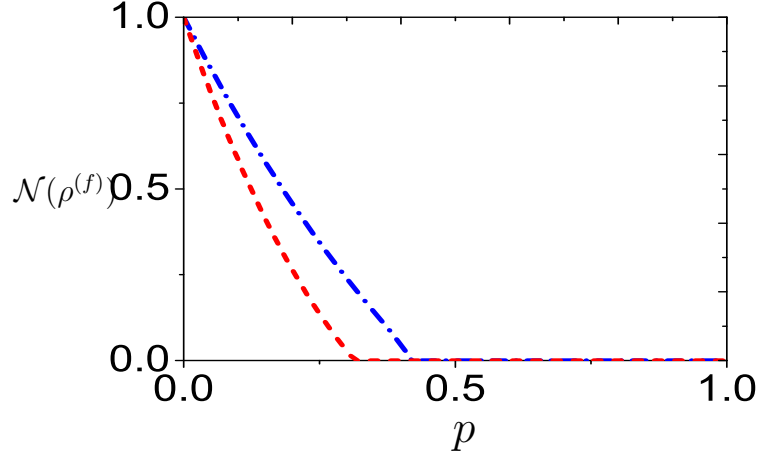


Figure 2: The dash-dot curves represents the entanglement  $\mathcal{N}(\rho^{(f)})$ , where it is assumed only two qubits are depolarized, while the dot curve when the 3-qubits are depolarized.

However, if we assume that the three qubits pass through the depolarized channel (2), then the final state (8) (for non-correlated noise) is given by

$$\begin{aligned}
 \rho_g^{(f_{nco3})} = & \mathcal{B}_1 (|000\rangle\langle 000| + |111\rangle\langle 111|) + \mathcal{B}_2 (|000\rangle\langle 111| + |111\rangle\langle 000|) \\
 & + \mathcal{B}_3 (|110\rangle\langle 110| + |001\rangle\langle 001| + |100\rangle\langle 100| + |011\rangle\langle 011|) \\
 & + \mathcal{B}_4 (|110\rangle\langle 001| + |010\rangle\langle 101| + |101\rangle\langle 010| + |100\rangle\langle 011| + |011\rangle\langle 100|) \\
 & + \mathcal{B}_5 (|001\rangle\langle 110| + \mathcal{B}_6 (|010\rangle\langle 010| + |101\rangle\langle 101|), \tag{15}
 \end{aligned}$$

where

$$\begin{aligned}
 \mathcal{B}_1 &= \frac{1}{2} \left[ (1-p)^3 + p(1-p) + \frac{5}{9}p^2(1-p) + \frac{p^3}{3} \right], \\
 \mathcal{B}_1 &= \frac{1}{2} \left[ (1-p)^3 - p(1-p) + \frac{5}{9}p^2(1-p) - \frac{p^3}{27} \right], \\
 \mathcal{B}_3 &= \frac{1}{3} \left[ p(1-p)^2 + \frac{7p^2}{3}(1-p) + \frac{p^3}{3} \right], \\
 \mathcal{B}_4 &= \frac{p^2}{9}(1-p), \quad \mathcal{B}_5 = \frac{p}{2}(1-p)^2 + \frac{p^2}{9}(1-p), \\
 \mathcal{B}_6 &= \frac{1}{3} \left[ p(1-p)^2 + \frac{5p^2}{3}(1-p) + \frac{p^3}{3} \right]. \tag{16}
 \end{aligned}$$

Fig.(2), shows the behavior of entanglement which is contained in the final states  $\rho_g^{(f_{nco3})}$  and  $\rho_g^{(f_{nco3})}$ , where it is assumed that the noise is non-correlated. The general behavior shows that the entanglement decays hastily as the channel parameter  $p$  increases. The decay rate of entanglement depends on the numbers of depolarized qubits. However, the decay rate of entanglement when the three qubit subject to the noise is larger than that depicted when only two qubits are depolarized. The

phenomena of the sudden-death of entanglement appear clearly when the particles are affected by non-correlated noise.

From Fig.(1) and Fig.(2), we can conclude that the decay of entanglement depends on the type of the noise (correlated or non-correlated) and the number of qubits which are subject to the noise. For correlated noise, the entanglement decays smoothly to reach its minimum bounds. These bounds depend on the number of polarized qubits, where they are non-zero for fewer number of polarized particles and completely vanish as the number of depolarized qubit increases. However, the phenomena of the sudden death appears in the presences of non-correlated noise and as the number of polarized qubit increases, the entanglement dies for smaller values of the channel parameter  $p$ .

## 4 W-state

Now we assume that the user share a three-qubits-state of W-type  $\rho_w = |\psi_w\rangle\langle\psi_w|$ , where  $|\psi_w\rangle$  is defined in (1). If we allow only one qubit to pass through the depolarized channel (3), then the final state is given by

$$\begin{aligned}\rho_w^{(f)} &= \frac{3-2p}{9} \left\{ |100\rangle\langle 100| + |010\rangle\langle 010| + |001\rangle\langle 010| + |001\rangle\langle 001| + |010\rangle\langle 001| \right\} \\ &+ \frac{3-4p}{9} \left\{ |100\rangle\langle 010| + |100\rangle\langle 001| + |010\rangle\langle 100| + |001\rangle\langle 100| \right\}.\end{aligned}\quad (17)$$

However, if we assume that only two qubits are forced to pass through the noise channel (3) and that the noise is correlated, then the final state is given by,

$$\begin{aligned}\rho_w^{f_{cor2}} &= \tilde{\mathcal{A}}_1 \left\{ |010\rangle\langle 010| + |010\rangle\langle 100| + |100\rangle\langle 010| + |100\rangle\langle 100| \right\} \\ &+ \tilde{\mathcal{A}}_2 \left\{ |010\rangle\langle 001| + |001\rangle\langle 100| + |001\rangle\langle 010| \right\} \\ &+ \tilde{\mathcal{A}}_3 \left\{ |100\rangle\langle 011| + |001\rangle\langle 001| \right\} + \tilde{\mathcal{A}}_4 |010\rangle\langle 101| + \tilde{\mathcal{A}}_5 |111\rangle\langle 111|,\end{aligned}\quad (18)$$

where

$$\begin{aligned}\tilde{\mathcal{A}}_1 &= \frac{(1-p)^2}{3} + \frac{P^2}{9}, \quad \tilde{\mathcal{A}}_2 = \frac{(1-p)^2}{3} - \frac{P^2}{27} \\ \tilde{\mathcal{A}}_3 &= \frac{(1-p)^2}{3} + \frac{P^2}{27}, \quad \tilde{\mathcal{A}}_4 = \frac{p^2}{27}, \quad \tilde{\mathcal{A}}_5 = \frac{2p^2}{27}\end{aligned}\quad (19)$$

However, if the three qubits pass through the non-correlated noise then the final state is given by

$$\begin{aligned}\rho_w^{(f_{co3})} &= \tilde{\mathcal{B}}_1 \left\{ |001\rangle\langle 001| + |001\rangle\langle 010| + |001\rangle\langle 100| \right\} + \tilde{\mathcal{B}}_2 |010\rangle\langle 001| + \tilde{\mathcal{B}}_3 |100\rangle\langle 011| \\ &+ \tilde{\mathcal{B}}_4 \left\{ |100\rangle\langle 100| + |100\rangle\langle 010| + |010\rangle\langle 010| + |010\rangle\langle 100| \right\} \\ &+ \tilde{\mathcal{B}}_5 \left\{ |010\rangle\langle 101| + |010\rangle\langle 111| + |010\rangle\langle 011| - |100\rangle\langle 111| - |111\rangle\langle 010| \right. \\ &\quad \left. - |111\rangle\langle 100| - |111\rangle\langle 111| \right\},\end{aligned}\quad (20)$$

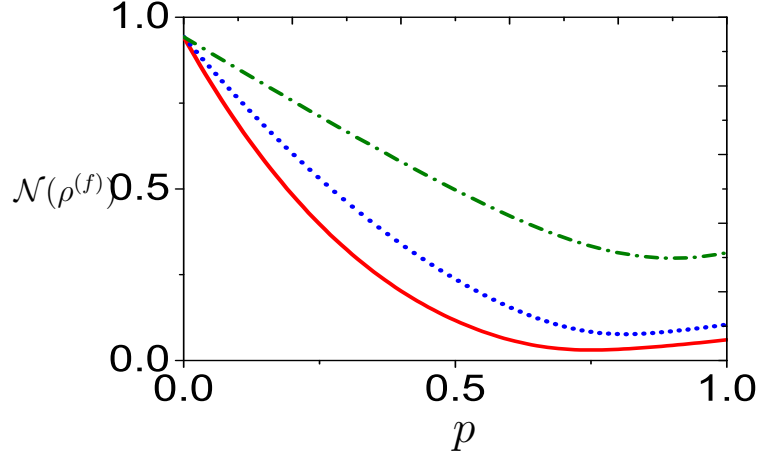


Figure 3: The dash-dot, dot and solid curves represent the entanglement of the final  $\rho_w^{(f)}$  when one, two and three qubits pass through the depolarized channel(3) in a correlated way, respectively.

where

$$\begin{aligned}\tilde{\mathcal{B}}_1 &= \frac{(1-p)^2}{3} + \frac{2p^2}{27}, & \tilde{\mathcal{B}}_2 &= \frac{(1-p)^2}{3} - \frac{p^2}{27}, \\ \tilde{\mathcal{B}}_3 &= \frac{(1-p)^2}{3} + \frac{p^2}{27}, & \tilde{\mathcal{B}}_4 &= \frac{(1-p)^2}{3} + \frac{2p^2}{9}, & \tilde{\mathcal{B}}_5 &= \frac{p^2}{27}.\end{aligned}\quad (21)$$

Fig.(3), describes the behavior of entanglement for the final states (16),(17) and (19), where we assume that, one, two and three qubits are subjected to the noise channel (3), respectively. The general behavior shows that the entanglement decays as the channel parameter  $p$  increases. The decay rate depends on the number of depolarized qubits. It is clear that, if only one qubit is depolarized, then the entanglement decays smoothly to reach its minimum non-zero value. These minimum values decrease as the number of depolarized qubits increases.

Now, we assume that the effect of the is non-correlated. If only two qubits are passing through the depolarized channel, then the noise output state is given by,

$$\begin{aligned}\rho_w^{fnc2} &= \tilde{\mathcal{A}}_1^{nc} \left\{ |000\rangle\langle 011| + |000\rangle\langle 101| + |011\rangle\langle 00| + |101\rangle\langle 000| \right\} \\ &+ \tilde{\mathcal{A}}_2^{nc} \left\{ |010\rangle\langle 010| + |100\rangle\langle 100| + |001\rangle\langle 001| \right\} + \tilde{\mathcal{A}}_3^{nc} |010\rangle\langle 100| \\ &+ \tilde{\mathcal{A}}_3^{nc} \left\{ |001\rangle\langle 010| + |001\rangle\langle 100| + |100\rangle\langle 001| + |010\rangle\langle 001| \right\} \\ &+ \tilde{\mathcal{A}}_4^{nc} \left\{ |110\rangle\langle 000| + |000\rangle\langle 110| \right\} + \tilde{\mathcal{A}}_5^{nc} |100\rangle\langle 010|,\end{aligned}\quad (22)$$

where

$$\begin{aligned}\tilde{\mathcal{A}}_1^{nc} &= \frac{2p}{27}, & \tilde{\mathcal{A}}_2^{nc} &= \frac{2}{27}p(1-p) + \frac{p^2}{27} + \frac{1}{3}, & \tilde{\mathcal{A}}_3^{nc} &= -\frac{2}{27}p(1-p) - \frac{5p^2}{27} + \frac{1}{3}, \\ \tilde{\mathcal{A}}_4^{nc} &= \frac{4p}{27}, & \tilde{\mathcal{A}}_5^{nc} &= \frac{p^2}{27} + \frac{1}{3}, & \tilde{\mathcal{A}}_6^{nc} &= \frac{2}{27}\{3p^2 - p\} + \frac{1}{3}.\end{aligned}\quad (23)$$

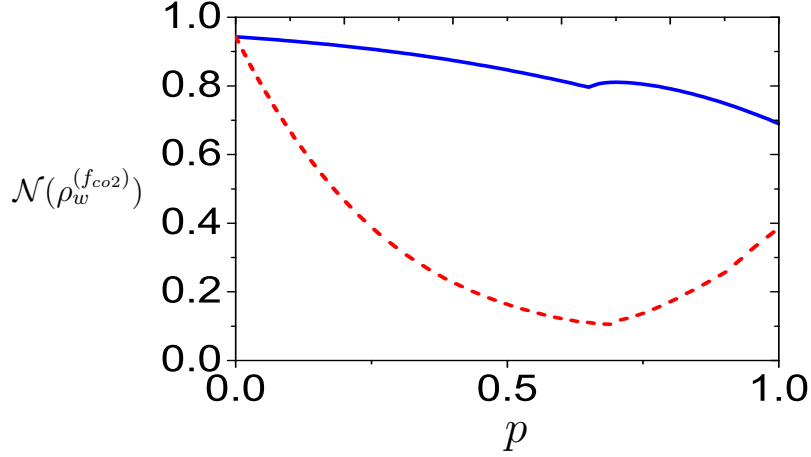


Figure 4: The solid and dash curves represent the entanglement of the final states  $\rho_w^{(f_{co2})}$  and  $\rho_w^{(f_{co3})}$ , respectively.

However, we assume that the three qubits of the W-states passing through the noise channel. In this case the output state is given by

$$\begin{aligned}
\rho_w^{(f_{nc3})} = & \tilde{\mathcal{B}}_1^{nc} \left\{ |110\rangle\langle 000| + |000\rangle\langle 110| + |101\rangle\langle 000| \right\} + \tilde{\mathcal{B}}_2^{nc} \left\{ |011\rangle\langle 000| + |000\rangle\langle 101| \right\} \\
& + \tilde{\mathcal{B}}_3^{nc} |111\rangle\langle 100| + \tilde{\mathcal{B}}_4^{nc} |011\rangle\langle 011| + \tilde{\mathcal{B}}_5^{nc} |101\rangle\langle 011| + \tilde{\mathcal{B}}_6^{nc} |000\rangle\langle 011| + \tilde{\mathcal{B}}_7^{nc} |010\rangle\langle 010| \\
& + \tilde{\mathcal{B}}_8^{nc} \left\{ |111\rangle\langle 111| + |110\rangle\langle 011| + |011\rangle\langle 100| + |011\rangle\langle 111| + |101\rangle\langle 101| \right\} \\
& + \tilde{\mathcal{B}}_9^{nc} \left\{ |001\rangle\langle 010| + |100\rangle\langle 010| + |010\rangle\langle 001| + |010\rangle\langle 100| \right\} \\
& + \tilde{\mathcal{B}}_{10}^{nc} \left\{ |100\rangle\langle 001| + |001\rangle\langle 100| \right\} + \tilde{\mathcal{B}}_{11}^{nc} \left\{ |001\rangle\langle 001| + |100\rangle\langle 100| \right\} \\
& + \tilde{\mathcal{B}}_{12}^{nc} \left\{ |011\rangle\langle 010| + |111\rangle\langle 010| + |110\rangle\langle 101| \right\}, \tag{24}
\end{aligned}$$

where

$$\begin{aligned}
\tilde{\mathcal{B}}_1^{nc} &= \frac{4}{3^2}p(1-p)^2 + \frac{1}{3^2}p^2(1-p) - \frac{4}{3^4}p^3, & \tilde{\mathcal{B}}_2^{nc} &= \frac{4}{3^2}p(1-p)^2 - \frac{4}{3^4}p^3, \\
\tilde{\mathcal{B}}_3^{nc} &= \frac{1}{3^2}p^2(1-p) + \frac{1}{3^4}p^3, & \tilde{\mathcal{B}}_4^{nc} &= \frac{2}{3^4}p^3, & \tilde{\mathcal{B}}_5^{nc} &= -\frac{1}{3^3}p^2(1-p) - \frac{2}{3^4}p^3, \\
\tilde{\mathcal{B}}_6^{nc} &= \frac{4}{3^2}p(1-p)^2 - \frac{2}{3^4}p^3, & \tilde{\mathcal{B}}_7^{nc} &= (1-p)^3 + \frac{4}{3^2}p(1-p)^2 + \frac{1}{3^2}p^2(1-p) + \frac{1}{3^4}p^3, \\
\tilde{\mathcal{B}}_8^{nc} &= \frac{1}{3^3}p^2(1-p), & \tilde{\mathcal{B}}_9^{nc} &= \frac{1}{3}(1-p)^3 - \frac{2}{3^2}p(1-p)^2 + \frac{1}{3^2}p^2(1-p) + \frac{5}{3^4}p^3, \\
\tilde{\mathcal{B}}_{10}^{nc} &= \frac{(1-p)^3}{3} + \frac{p^2}{3^2}(1-p) + \frac{5p^3}{3^4}, & \tilde{\mathcal{B}}_{11}^{nc} &= \frac{(1-p)^3}{3} + \frac{2p}{3^2}(1-p)^2 + \frac{p^2}{3^2}(1-p) + \frac{p^3}{3^4}, \\
\tilde{\mathcal{B}}_{12}^{nc} &= -\frac{p^2}{3^3}(1-p). \tag{25}
\end{aligned}$$

The behavior of entanglement for the output states (21) and (23) is described in Fig.(4). In general, the entanglement decreases as the channel's strength increases. However the decreasing rate is smaller than that depicted for correlated noise (see Fig.3).



Moreover, when all three qubits pass through the depolarized channel, the entanglement decays faster than that depicted for two depolarized qubits. It is clear that for larger values of the strength parameter  $p$ , the entanglement increases. However, the upper bound of entanglement is smaller than that in the case of two-depolarized qubit.

On the other hand, comparing the behavior of this entanglement with that for the GHZ state, we can see that the W-state is more robust than the GHZ state, where for the GHZ state the entanglement completely vanishes in a sudden way for smaller values of the channel strength.

## 5 Conclusion

In this contribution, we investigate the behavior of entanglement for two classes of multi-qubit states; the GHZ and W-states passing through the depolarized channel. In our treatment, we assume that the effect of the noise channel includes two possibilities. In the first possibility we assume that the qubits are subject to the same noise at the same time (correlated noise). The second possibility, which is called non-correlated noise, the particles are subjected to different noises at the same time. Analytical expressions are obtained for the final output states. For the correlated noise, we calculate the final output states when one, two or three qubits are forced to pass through the depolarized channel. However, for the non-correlated noise, we consider the case of two or three qubits passing through the depolarized channel.

The dynamics of the survival amount of entanglement is investigated against the channel strength. The general behavior shows that the entanglement decreases as the channel parameter increases. However, the decreasing rate depends on the number of depolarized qubits, the type of the noise and the type of the initial state. For *correlated* noise, the decay rate of entanglement decreases as the number of polarized qubits increases. Although the decay rate for the W-state is larger than that depicted for the GHZ state, the entanglement completely vanishes when the three qubits are depolarized for the GHZ state. On the other hand, for the *non-correlated* noise, the phenomena of the sudden death of entanglement appears when the GHZ's qubits are depolarized. If only two qubit qubits are depolarized, the entanglement vanishes at larger values of the channel parameter. However, for the W-state, the phenomena of the entanglement sudden death doesn't appear and the lower bound of entanglement is large.

*In conclusion*, the robustness or fragility of the initial states which pass through a depolarized channel depends on the type of the noise: correlated or non-correlated. In general GHZ is more fragile than W-state. The phenomena of entanglement sudden death appears only for non-correlated noise.

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